# Data Structures
# CSCI H343, Fall 2021

**Midterm**

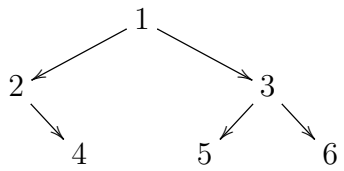
**Name:** _____

This exam has 11 questions, for a total of 100 points.

1. [8 points] What is the output of this Java program?

```
public class Node {
    Node left, right; int data;
    Node(int d) { data = d; }
    public void f() {
        System.out.print("(");
        if (left != null) {
            left.f();
            System.out.print("_");
        }
        if (right != null) {
            right.f();
            System.out.print("_");
        }
        System.out.print(data);
        System.out.print(")");
    }

    public static void main(String[] args) {
        Node a = new Node(0);
        Node b = new Node(1);
        Node c = new Node(2);
        Node d = new Node(3);
        a.left = b;
        a.right = c;
        c.left = d;
        a.f();
    }
}
```

2. [8 points] Consider this binary tree.




The following questions are with respect to an inorder traversal.

1. Which node comes immediately after node 1?
2. Which node comes immediately after node 5?
3. Which node comes immediately before node 5?
4. Which node comes immediately before node 1?

3. [10 points] For the following `Node` class in a binary tree, Fill in the blanks to complete the following implementation of the `next` method that returns the node that comes after the current node with respect to an inorder traversal, if there is one, and `null` if there is none.

```
class Node {
    T data;
    Node left, right, parent;
    Node next() {
        if (right == null) {
            return ___(a)___;
        } else {
            return ___(b)___;
        }
    }
    Node first() {
        if (left == null) {
            return this;
        } else {
            return ___(c)__;
        }
    }
    Node nextAncestor() {
        Node n = this, p = parent;
        while (p != null && ___(d)___) {
            n = p;
            p = ___(e)___;
        }
        return p;
    }
}
```

4. [8 points] The `divide` function is meant to divide integer $m$ by the integer $n$, returning the quotient $q$ and remainder $r$. The integers $m$ and $n$ are required to be non-negative. The correctness criteria for `divide` is that the quotient $q$ and remainder $r$ should satisfy

$$m = nq + r \qquad 0 \leq r < n$$

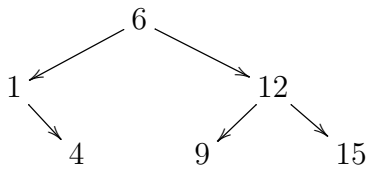
```
Pair<Integer, Integer> divide(int m, int n) {
    int q = 0;
    int r = m;
    while (r >= n) {
        r = r - n;
        q = q + 1;
    }
    return new Pair<>(q, r);
}
```

1. State the loop invariant for the `while` loop.
2. Explain why the loop invariant is true before the start of the loop.
3. For a hypothetical iteration of the loop, explain why the loop invariant is true at the end of the loop body, assuming only that the loop invariant was true at the beginning of the loop body.
4. Explain why the loop invariant combined with the loop condition being false logically implies the correctness criteria for the `divide` function.

5. [12 points] What is the big-O time complexity of the following `flood` method in terms of the total number of tiles, represented by $n$? Provide an argument for your answer that analyzes every statement in the method and how their individual time complexities combine into the total time complexity.

```
public static void flood(WaterColor color,
                         LinkedList<Coord> flooded_list,
                         Tile[][] tiles,
                         Integer board_size) {
    HashSet<Coord> flooded_set = new HashSet<>(flooded_list);
    ArrayList<Coord> flooded_array = new ArrayList<>(flooded_list);
    for (int i = 0; i != flooded_array.size(); ++i) {
        Coord c = flooded_array.get(i);
        for (Coord n : c.neighbors(board_size)) {
            if (!flooded_set.contains(n)
                  && tiles[n.getY()][n.getX()].getColor() == color) {
                flooded_array.add(n);
                flooded_list.add(n);
                flooded_set.add(n);
            }
        }
    }
}
```

6. [7 points] Draw the result of inserting key 8 into the following Binary Search Tree.



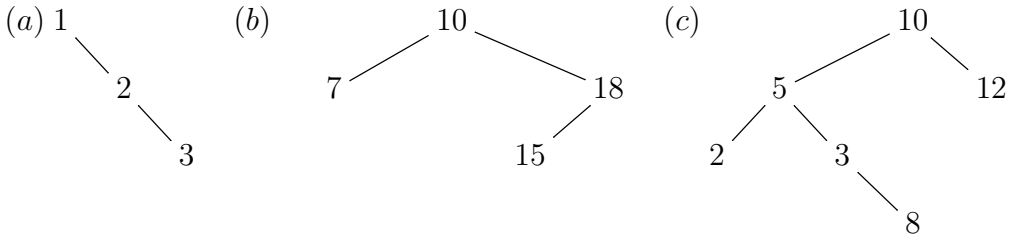
7. [10 points] Write the Java code for the implementation of the below `find_first_equal` function. Recall that the `Iterator` interfaces is defined as follows.

```
interface Iterator<T> {
     T get();
     void set(T e);
     void advance();
     void advance(int n);
     boolean equals(Iterator<T> other);
     Iterator<T> clone();
}
```

The `find_first_equal` function returns an iterator pointing to the first element in the half-open range `[begin,end)` that equals the 'x' parameter. If no element equals 'x', return the `end` iterator. The `begin` and `end` iterators must not be changed.

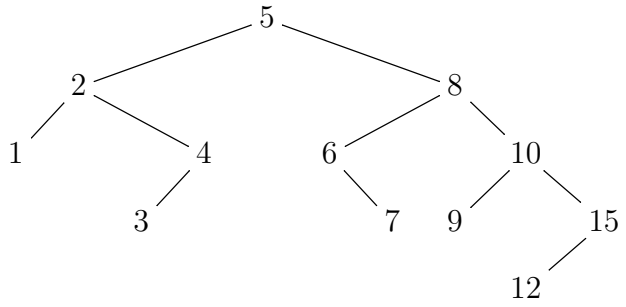
```
public static <E> Iterator<E>
find_first_equal(Iterator<E> begin, Iterator<E> end, E x) {









}
```

8. [12 points] Which of the following trees are binary search trees?
Which of them are AVL trees?



9. [8 points] Show that $3n + n \log_2 n \in O(n^2)$ using the definition of big-O.

10. [8 points] Given the following AVL binary search tree, remove key 5, maintaining the binary search tree and AVL properties. Explain each change that you make to the tree and draw the tree after each change.



11. [9 points] What is the big-O time complexity of the following anagram detection function in terms of the sum $n$ of the lengths of the two input strings? Provide an argument for your answer that analyzes each method and loop and how their individual time complexities combine into the total time complexity.

```
private static LinkedList<Character> copy_without_spaces(String s) {
    LinkedList<Character> c = new LinkedList<>();
    for (int i = 0; i != s.length(); ++i) {
        if (s.charAt(i) != ' ') {
            c.add(s.charAt(i));
        }
    }
    return c;
}
public static boolean find_remove(LinkedList<Character> l, Character c) {
    Iterator<Character> iter2 = l.iterator();
    while (iter2.hasNext())
        if (iter2.next() == c) {
            iter2.remove();
            return true;
        }
    return false;
}

public static boolean anagram(String s1, String s2) {
    LinkedList<Character> l1 = copy_without_spaces(s1);
    LinkedList<Character> l2 = copy_without_spaces(s2);
    for (Character c1 : l1) {
        if (! find_remove(l2, c1))
            return false;
    }
    return l2.size() == 0;
}
```